

**DEPARTMENT OF MATHEMATICS**  
**M. Phil Mathematics**  
**M. Phil PROGRAMME**  
**REVISED SYLLABUS**



**Loyola College (Autonomous)**  
**Chennai- 600 034**

**Department of Mathematics**  
**Loyola College, Chennai-600034**

**M. Phil Restructuring**  
**2018-2019**

Semester	Subject Code	Paper No	Paper Title	T/L/P	Hours	Credits
I		1	RESEARCH METHODOLOGY	T	3	3
		2	ALGEBRA AND ANALYSIS	T	6	6
II		3A	ADVANCED ASTRONOMY	T	3	3
		3B	ADVANCED NUMBER THEORY AND CRYPTOGRAPHY	T	3	3
		3C	ADVANCED GRAPH THEORY	T	3	3
		3D	DIFFERENTIAL TOPOLOGY	T	3	3
		3E	OPTIMIZATION TECHNIQUES	T	3	3
		3F	GEODESIC CONVEXITY IN GRAPHS	T	3	3
		3G	FUZZY SUBSETS AND APPLICATIONS	T	3	3

**Department of Mathematics**  
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**PAPER 1:**

**RESEARCH METHODOLOGY**

**Objectives**

- Students will be introduced to understand the different processes and parameters of research.
- Students will be guided to improve the quality of research report and thesis presentation.

**Outcomes**

- Students will be able to carry out the research problem individually in a perfect scientific method and skill in LATEX at the end of the course.

**Unit 1:**

**Research Definition and Formulation:** Research Definition-Types of Research-Objectives and Strategies of Research-Variou Steps in Research Process-Qualities of Researcher-Components of a Research Problem-Formulation of Research Problem-Research Design.

**Unit 2:**

**Data Collection:** Data Sources: Primary, Secondary and Tertiary Data-Literature Survey- Interpretation of data-Questionnaire Designing-Survey and Case Study Research-Laboratory Research-Critical Review- Data Analysis.

**Unit 3:**

**Quantitative Methods:** Probabilistic and Non-Probabilistic Sampling-Sampling Errors: Type I Error and Type II Error-Estimation-Hypothesis Testing and Applications-Standard Deviation-Normal Distribution, Properties-Correlation and Regression Analysis.

**Unit 4:**

**Data Interpretation and Research Tools:** Layout of a Research Paper-Impact Factor of Journals-Ethics in Publication-Plagiarism and Self-Plagiarism- Software for LATEX Paper Formatting- Software for Detection of Plagiarism.

**Unit 5:**

**Research Reports:** Structure and Components of Research Report-Types of Report-Mechanism of writing a report-Layout, structure and language of a typical report-Organization of the research report- Bibliography, referencing and footnotes-Thesis writing.

**REFERENCES**

1. C.R. Kothari, Research Methodology-Methods and Techniques, New Wiley Eastern Ltd., Delhi, 2009.
2. A.K. Jain, Ethical issues in scientific publication, Indian Journal of Orthopaedics, Vol. 44, no. 3, 235-237, 2010.
3. J. Stegmann, How to evaluate journal impact factors, Nature, Vol. 390, 550, 1997.
4. M. Roig, Plagiarism and self-plagiarism: What every author should know, BiochemiaMedica, Vol. 20, no. 3, 295-300, 2010.

## PAPER 2

### ALGEBRA AND ANALYSIS

#### **Objective:**

- Students will be introduced to know the advanced topics in algebra and analysis.

#### **Outcome:**

- The students will be capable of handling the advanced topics in algebra and analysis.

#### **Unit1:**

**Modules:** Basic Definitions – Quotient Modules – Module Homomorphisms – Generation of Modules – Direct Sums – Free Modules.

#### **Unit2:**

**Structure of Modules:** Change of scalars – Simple Modules – Semi-simple Modules – Structure of Semi-simple Modules – Chain conditions – The Radical.

#### **Unit3:**

**Abstract Integration:** The concept of measurability – Simple functions – Elementary properties of measures – Integration of positive functions – Integration of complex functions – The role played by the set of measure zero.

#### **Unit4:**

**Elementary Properties of Holomorphic Functions:** Complex differentiation – Integration over paths – The local Cauchy theorem – The power series representation – The global Cauchy theorem – The calculus of residues.

#### **Unit5:**

**Fourier Transforms:** Formal properties – The inversion theorem – The Plancherel Theorem – The Banach algebra  $L^1$ .

#### **BOOKS FOR STUDY:**

1. Pierce R.S., "Associative Algebras, Graduate Texts in Mathematics", Springer Verlag, New York, 1982.
2. Rudin W., "Real and Complex Analysis", Tata Mc-Graw Hill, Third Edition, 2006.

#### **REFERENCES:**

1. Serge Lang, "Algebra", Addison Wesley, Third Edition, 2005.
2. Paul R. Halmos, "Measure Theory", Springer 1974.
3. Royden, H.L., "Real Analysis", The Macmillan Company, New York, Third Edition, 1988.
4. Elras, M. Stein and Ramishakarchi, "Complex Analysis", Princeton University Press, 2003.

**GUIDE SPECIAL PAPER**  
**PAPER 3(A)**

**ADVANCED ASTRONOMY**

**Objective:**

- Students will be introduced to the concept of cosmology and stars.

**Outcome:**

- Students will be able to lead research in theoretical astronomy.

**Unit 1:**

Constellations and nomenclature of stars– The cardinal points and circles on the celestial sphere– Equatorial, ecliptic and galactic system of co-ordinates– Spherical triangle and related problems– Aspects of sky from different places on the earth– Twilight, Seasons, Sidereal, Apparent and Mean solar time and their relations– Equation of time– Ephemeris and Atomic Times– Calendar– Julian date and heliocentric correction.

**Unit 2:**

Utility of stellar spectrum– basic knowledge of stellar atmospheres– Binaries– variable stars– clusters– open and globular clusters– Laws of planetary motion– Motions and Distances of Stars– Statistical and moving cluster parallax.

**Unit 3:**

Effects of atmospheric refraction– aberration– parallax– precession– nutation and proper motion on the coordinates of stars– Reduction from apparent to mean places and vice versa.

**Unit 4:**

Distances of stars from the trigonometric– secular and moving cluster parallaxes– Stellar motions– Magnitude scale and magnitude systems– Atmospheric extinction– Absolute magnitudes and distance modulus– color index– Black-body approximation to the continuous radiation and temperatures of stars– Variable stars as distance indicators.

**Unit 5:**

Visual– spectroscopic and eclipsing binaries– Importance of binary stars as source of basic astrophysical data– Classification and properties of various types of intrinsic and eruptive variable stars– Astrophysical importance of the study of variable stars– Novae and Supernovae.

**Books for Study:**

1. Harwit, M. Astrophysical Concepts, 3rd ed, Springer-verlag, 2006.
2. K.D.Abhyankar: Astrophysics: Stars and Galaxies. Tata McGraw Hill Publication (Chap.2)
3. Padmanabhan, T., Theoretical Astrophysics, Vols.1-3, Cambridge University Press, 2005.

**References:**

1. W.M.Smart: Text book of Spherical Astronomy.
2. A.E.Roy: Orbital Motion.
3. McCusky: Introduction to Celestial Mechanics.
4. K.D.Abhyankar: Astrophysics: Stars and Galaxies. Tata McGraw Hill Publication (Chap.2)
5. G.Abell: Exploration of the Universe.
6. A.Unsold: New Cosmos.
7. BaidyanathBasu: Introduction to Astrophysics.

**GUIDE SPECIAL PAPER**  
**PAPER 3(B)**

**ADVANCED NUMBER THEORY AND CRYPTOGRAPHY**

**Objective:**

- Students will be introduced to know the fundamentals of number theory and cryptography such as congruence's, residues and partitions.

**Outcome:**

- Students will be able to understand and apply the concepts in solving problems of cryptosystems.

**Unit1:**

**CONGRUENCES:** Congruences– Solutions of congruences– congruences of degree 1– The function  $\phi(n)$ – Congruences of higher degree– Prime power modulo– Prime modulus– congruences of degree 2– Prime modulus– Power residues.

**Unit2:**

**QUADRATIC RESIDUES:** Quadratic residues– Quadratic reciprocity– The Jacobi symbol– greatest integer function– arithmetic function– The Mobius Inversion formula– The multiplication of arithmetic functions.

**Unit3:**

**DIOPHANTINE EQUATIONS:** Diophantine equations– The equation  $ax+by=c$ – Positive solutions– Other linear Equations– Sums of four and five squares– Waring's problem– sum of fourth powers– Sum of two Squares.

**Unit4:**

**TRADITIONAL SYMMETRIC-KEY CIPHERS:** Substitution Ciphers– Transposition Ciphers– Steam and Block Ciphers– Modern Block Ciphers– Modern Steam Ciphers– DES– AES.

**Unit5:**

**ASYMMETRIC KEY CRYPTOGRAPHY:** RSA Cryptosystem– Rabin Cryptosystem– ElGamal Cryptosystem– Elliptic Curve Cryptosystem.

**BOOKS FOR STUDY:**

1. Ivan Niven, Herbert S. Zuckermann, Hugh L. Montgomery, "An Introduction to the Theory of Numbers", John Wiley, Fifth Edition, 2006.
2. Behrouz A. Forouzan, "Cryptography & Network Security", Tata McGraw Hill, Special Indian Edition, 2007.

**REFERENCES:**

1. Tom M. Apostol, "Introduction to analytic number theory", Narosa Publishing House, 1980.
2. Rose H.E., "A Course in Number Theory", Clarendon Press, Second Edition, 1995.
3. Kenneth Ireland & Michael Rosen, "A Classical Introduction to Modern Number Theory", Springer International Edition, Second Edition, 2010.
4. Koblitz, N., "A course in number theory and cryptography", Springer Verlag, 1994.
5. Stinson D.R., "Cryptography: Theory and Practice", CRC Press, Third Edition, 2002.

**GUIDE SPECIAL PAPER**  
**PAPER 3(C)**

**ADVANCED GRAPH THEORY**

**Objective:**

- Students will be introduced advanced topics in graph theory.

**Outcome:**

- Students will be able to pursue research in frontier areas of graph theory.

**Unit1:**

**CONNECTIVITY IN GRAPHS:** Vertex connectivity–Edge connectivity–Blocks–k-connected and k-edge connected graphs–Network flow problems.

**Unit2:**

**COLORING OF GRAPHS:** Vertex colorings and upper bounds–Brooks’ theorem–Graphs with large chromatic number–Turan’s theorem–Counting proper colorings–Edge colouring–Characterization of line graphs.

**Unit3:**

**PLANAR GRAPHS:** Embeddings and Euler’s formula–Dual graphs–Kuratowski’s theorem – 5 colour theorem–Crossing number–Surface of higher genus.

**Unit4:**

**RAMSEY THEORY:** The pigeonhole principle–Ramsey’s theorem–Ramsey numbers–Graph Ramsey theory.

**Unit5:**

**EIGENVALUES OF GRAPHS:** The characteristic polynomial–Linear algebra of real symmetric matrices–Eigenvalues and graph parameters – Eigenvalues of regular graphs–Strongly regular graphs.

**BOOKS FOR STUDY:**

1. Douglas B. West, “Introduction to Graph Theory”, Prentice Hall of India, Second Edition, 2002.

**REFERENCES:**

1. Bondy J.A, and Murty U.S. R., “Graph Theory”, Springer, 2008.
2. Balakrishnan R. and Ranganathan K., “A text book of Graph Theory”, Springer, 2012.
3. Graham R.L., Rothschild B.L and Spencer J.H., “Ramsey Theory”, Wiley Publishers, Second Edition, 1990.
4. Biggs N., “Algebraic Graph Theory”, Cambridge Tracts in Mathematics 67, Cambridge University Press, 1994.

**GUIDE SPECIAL PAPER**  
**PAPER 3(D)**

**DIFFERENTIAL TOPOLOGY**

**Objective:**

- Students will be introduced to the notion of smooth manifolds and classify compact manifolds and smooth compact surfaces.

**Outcome:**

- Students will be able to solve the problems in differential manifolds occur in different fields like mathematics, physics, mechanics and economics.

**Unit1:**

**MANIFOLDS AND MAPS:** Derivatives and tangents–inverse function theorem and immersions–submersions–homotopy and stability–Sard’s theorem and Morse functions–embedding manifolds in Euclidean space.

**Unit2:**

**TRANSVERSALITY AND INTERSECTION:** Manifolds with boundary–one manifold and some consequences–transversality–intersection theory modulo 2–winding numbers and the Jordan–Brouwer separation theorem.

**Unit3:**

**ORIENTED INTERSECTION THEORY:** Orientation on manifolds–oriented intersection number–degrees of maps–fundamental theorem of algebra–Euler characteristic as an intersection number.

**Unit4:**

**APPLICATIONS OF INTERSECTION THEORY:** Lefschetz Fixed point theory – Borsuk-Ulam theorem – vector fields – isotopy – Hopf degree theorem.

**Unit5:**

**COMPACT SMOOTH SURFACES:** Morse functions–Morse Lemma–Connected sum–Attaching handles–Handle decomposition theorem–Application to smooth classification of compact smooth surfaces.

**BOOKS FOR STUDY:**

1. Guillemin V. and Pollack A., “Differential Topology”, Prentice-Hall, 1974.
2. Morris W. Hirsch, “Differential topology”, Springer-Verlag, 1976.

**REFERENCES:**

1. Milnor J., “Topology from the differentiable viewpoint, Princeton Landmarks in Mathematics”, Princeton University Press, 1997.
2. Shastri A.R., “Elements of Differential Topology”, CRC Press, 2011.



**GUIDE SPECIAL PAPER**  
**PAPER 3(E)**

**OPTIMIZATION TECHNIQUES**

**Objective:**

- Students will be introduced to various operations research tools in decision making in an organization.

**Learning outcome:**

- Students will be capable of using advanced techniques in various operations research tools in decision making and able to formulate organization problems into operations research models for seeking optimal solutions.

**Unit1:**

**ADVANCED LINEAR PROGRAMMING AND GOAL PROGRAMMING:** Revised simplex algorithm – Dantzig – Wolfe decomposition algorithm – Karmarkar interior point algorithm – Goal programming algorithms.

**Unit2:**

**HEURISTIC PROGRAMMING:** Greedy Heuristics: Discrete variable heuristic – Continuous variable heuristic – Metaheuristic: Tabu search algorithm – Simulated annealing algorithm – Genetic algorithm.

**Unit3:**

**DECISION ANALYSIS:** Decision making under certainty – Hierarchy process – Decision making under risk – Decision tree based expected value criterion – Variants of the expected value criterion – Decision making under uncertainty.

**Unit 4:**

**NON-LINEAR PROGRAMMING:** Unconstrained algorithms: Direct search method – Gradient method – Constrained algorithms: Separable programming, Chance-constrained programming.

**Unit 5:**

**SIMULATION:** Nature and need for simulation – Monte-Carlo simulation – Generation of pseudo random numbers by mid-square method, Congruence multiplier method – Test for randomness – Generating random variates for Uniform, Exponential, Erlangian, Poisson, Normal distributions – Application to simple problems in operations research.

**BOOKS FOR STUDY:**

1. Hamdy A. Taha, "Operations Research - An Introduction", Pearson Education, New Delhi, Tenth Edition, 2018.
2. Geoffrey Gordon, "System Simulation", Pearson Education, New Delhi, Second Edition, 2017.

**REFERENCES:**

1. Harvey M. Wagner, "Principles of Operations Research with Applications to Managerial

- Decisions”, Prentice-Hall of India Pvt. Ltd., New Delhi, Second Edition, 1975.
2. Rao S.S., “Engineering Optimization: Theory and Practice”, Wiley and New Age International, Fourth Edition, 2009.
  3. Sharma J K ., “Operations Research”, Trinity Press, New Delhi, Sixth Edition, 2017.

**GUIDE SPECIAL PAPER**  
**PAPER 3(F)**

**GEODESIC CONVEXITY IN GRAPHS**

**Objective:**

- Students will be introduced to the geodesic convexity ideas in graph theory and real life problem.

**Learning outcome:**

- The students will be able to apply geodesic convexity concepts in artificial intelligence, parallel interconnection network etc.,

**Unit 1:**

Introduction-Graph Theory- Metric Graph Theory-Convexity Spaces-Graph Convexities.

**Unit 2:**

Invariants-Geodetic Closure And Convex Hull-Geodetic And Hull Numbers-  
Monophonic And M-Hull Numbers-Convexity Number-Forcing Geodomination

**Unit 3:**

Closed Geodomination-Geodetic Domination-K-Geodomination-Edge Geodomination-  
Classical Parameters.

**Unit 4:**

Graph Operations-Cartesian Product-Strong Product-Lexicographic Product-Join-Corona  
Product.

**Unit 5:**

Oriented Graphs-Computational Complexity

**Text Book:**

Geodetic Convexity In Graphs- Ignaciom.Pelayo-Springer.

**GUIDE SPECIAL PAPER**  
**PAPER 3(G)**

**FUZZY SUBSETS AND ITS APPLICATIONS**

**Objectives:**

- Students will be able to understand the mathematical concepts that help in analyzing the uncertain situations.
- Students will be able to recognize mathematical tools to apply to real life situations.

**Learning outcomes:**

- Students will be able to get a deeper analysis on how mathematical concepts are extended into fuzzy world of uncertainties.
- Students will be able to equip themselves with fuzzy theoretical tools and also application nuances.

**Unit 1:**

Introduction – Characteristic Function – Membership Function – Characteristics and significance of the paradigm shift – The concept of a fuzzy subset – Crisp set vs fuzzy subsets – Operations on fuzzy subsets – Properties of the set of the fuzzy subsets – Types of fuzzy sets – Extension principle.

**Unit 2:**

Berge graph – Fuzzy graphs – Fuzzy relations – Fuzzy subsets induced by a mapping – Conditioned fuzzy subsets – Properties of fuzzy binary relation – Transitive closure of a fuzzy binary relation – Fuzzy preorder relations – Similitude relations – Similitude sub relations in a fuzzy preorder – Antisymmetry – Fuzzy order relations – Ordinal relations – Dissimilitude relations – Resemblance relations – Dissemblance relation.

**Unit 3:**

Classical logic – Multivalued logic – Fuzzy logic – Truth tables and linguistic approximation – Linguistic variables – Linguistic hedges – Fuzzy propositions – Types of fuzzy propositions – Fuzzy quantifiers – Fuzzy inference rules – Fuzzy control system.

**Unit 4:**

Fuzzy arithmetic – Interval of confidence – Fuzzy numbers – Operations on fuzzy numbers – Maximum and minimum of fuzzy numbers – L-R fuzzy number – Triangular fuzzy number – Trapezoidal fuzzy number – Quadrilateral fuzzy number – Pentagonal fuzzy number – Operations on triangular, trapezoidal and pentagonal fuzzy numbers – Interval of confidence with fuzzy numbers of type-2 – Functions of fuzzy numbers – Derivative of a function of fuzzy numbers.

**Unit 5:**

Decisions – Fuzzy Decisions – Individual decision making – Group or Multiperson decision making – Fuzzy multi criteria decision making – Fuzzy multi objective decision making – Fuzzy multi attribute decision making – Fuzzy analytic hierarchy process – Fuzzy TOPSIS – Applications in medicine, socio-economic studies, genetics – engineering and robotics.

**Text Book:**

1. A. Kaufmann, Introduction to the Theory of Fuzzy Subsets – Volume 1, Academic Press, New York, 1975.  
[Chapter: Sections 1 – 8, 10 - 26]
2. George J. Klir and Bo Yuan, Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice Hall of India, New Delhi, 2002.  
[Chapter: 8; Sections 8.1 – 8.8, Chapter: 15; Sections 15.1-15.4]
3. A. Kaufmann and M. M. Gupta, Introduction to Fuzzy Arithmetic: Theory and Applications, International Thomson Computer Press, Boston, U.S.A, 1991.  
[Chapter: 1; Sections 1.1 – 1.6, 1.9-1.10, Chapter: 2; Sections 2.7, Chapter: 3; Sections 3.6-3.7]
4. H. J. Zimmermann, Fuzzy Set Theory and its Applications, Kluwer Academic Publishers, The Netherlands, 1992.  
[Chapter: 3; Sections 3.1, Chapter: 5; Sections 5.1, Chapter: 9; Sections 9.1-9.2, Chapter: 12; Sections 12.1, 12.4]
5. A. Emrouznejad and William Ho, Fuzzy Analytic Hierarchy Process, CRC Press, Taylor & Francis Group, Boca Raton, FL, 2018.  
[Chapter: 1, 2, 6]

**References:**

1. Lotfi A. Zadeh, Fuzzy Sets and their Applications to Cognitive and Decision Processes, Academic Press, New York, 1975.
2. Bart Kosko, Neural Networks and Fuzzy Systems, Prentice-Hall of India, New Delhi, 2003.
3. T. L. Saaty and L. G. Vargas, Models, Methods, Concepts & Applications of the Analytic Hierarchy Process, Springer Science+Business Media, LLC, New York, 2001.
4. Ching-Lai Hwang and Kwangsun Yoon, Multiple Attribute Decision Making: Methods and Applications, Springer-Verlag, Berlin Heidelberg, 1981.

**GUIDE SPECIAL PAPER**  
**PAPER 3(H)**

**FUZZY LOGIC AND ITS APPLICATIONS**

**Objectives:**

Students will be able to

- Fundamentals of fuzzy logic
- Concepts of fuzzy sets, knowledge representation using fuzzy rules, fuzzy logic, fuzzy prepositions and fuzzy quantifiers and applications of fuzzy logic.
- Conceptualize the principle, formulation and computational procedure of fuzzy logic
- Recognize fuzzy logic membership function and fuzzy inference systems.
- Make applications on Fuzzy logic membership function and fuzzy inference systems.

**COURSE OUTCOMES:**

CO1: familiarizing the basic of fuzzy concepts

CO2: exploring the techniques involved in fuzzy logic.

CO3: enhancing the skill of artificial neural network

CO4: Explore the functional components of of fuzzy logic classification on controller.

CO5: Develop and implement a basic trainable fuzzy logic system to design and manufacturing.

**UNIT I : INTRODUCTION TO FUZZY LOGIC PRINCIPLES**

Basic concepts of fuzzy set theory – operations of fuzzy sets – properties of fuzzy sets – Crisp relations – Fuzzy relational equations – operations on fuzzy relations – fuzzy systems – propositional logic – Inference – Predicate Logic – Inference in predicate logic – fuzzy logic principles – fuzzy quantifiers – fuzzy inference – fuzzy rule based systems – fuzzification and defuzzification – types.

**UNIT II : ADVANCED FUZZY LOGIC APPLICATIONS**

Fuzzy logic controllers – principles – review of control systems theory – various industrial applications of FLC adaptive fuzzy systems – fuzzy decision making – Multiobjective decision making – fuzzy classification – means clustering – fuzzy pattern recognition – image processing applications – systactic recognition – fuzzy optimization.

**UNIT III : INTRODUCTION TO ARTIFICIAL NEURAL NETWORKS**

Fundamentals of neural networks – model of an artificial neuron – neural network architectures – Learning methods – Taxonomy of Neural network architectures – Standard back propagation algorithms – selection of various parameters – variations Applications of back propagation algorithms.

#### **UNIT IV : OTHER ANN ARCHITECTURES**

Associative memory – exponential BAM – Associative memory for real coded pattern pairs – Applications adaptive resonance theory – introduction – ART 1 – ART2 – Applications – neural networks based on competition – kohonenself organizing maps – learning vector quantization – counter propagation networks – industrial applications.

#### **UNIT V : RECENT ADVANCES**

Fundamentals of genetic algorithms – genetic modeling – hybrid systems – integration of fuzzy logic, neural networks and genetic algorithms – non traditional optimization techniques like ant colony optimization – Particle swarm optimization and artificial immune systems – applications in design and manufacturing.

#### **TEXT BOOKS:**

- Klir.G, Yuan B.B. “Fuzzy sets and Fuzzy Logic Prentice Hall of India private limited, 1997.
- LauranceFausett, “Fundamentals of Neural Networks”, Prentice hall, 1992
- Rajasekaran. S..VijayalakshmiPai. G.A. “Neural Networks, Fuzzy Logic and Genetic Algorithms”, Prentice Hall of India Private Limited, 2003
- Timothy J.Ross, “Fuzzy logic with Engineering Applications”, McGraw Hill, 1995

#### **REFERENCES:**

- Gen, M. and Cheng R. “Genetic Algorithm and Engineering Design”, john wiley 1997
- Zurada J.M. “Introduction to Artificial Neural Systems”, Jaico publishing house, 1994.